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# Control of supply in water networks – linear case

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Methodology (based on so-called Dynamic Virtual Distortion Method) of control of water supply in water networks is presented. Minimization of supply pressure in inlets to the network, subject to inequality constraints imposed on outlet pressure (in chosen nodes) is discussed. Taking advantage of pre-computed influence vectors, the real-time control strategy can be realised with small computational effort and therefore, can be managed with use of hardware-based controllers. Linear constitutive relation (water flow vs. drop of pressure) has been assumed in order to develop the first step of the methodology. However, generalisation of the proposed approach for a piece-wise-linear case is planned in the near future.

Key words: *water networks, optimal supply control, VDM based design.*

## 1. Introduction

A software tool for signal processing in control of supply in water networks is presented. It is assumed that the water pressure in the network's nodes in a distance of controlled inlet can be measured and also that the inlet pressure can be modified in real time in a controlled way. Then, making use of the analytical network model of this installation and using presented below, so called Virtual Distortion Method (VDM), the control of water supply can be performed.

The problem of management of water sources is more and more important in the world scale. Therefore, there is requirement for an automatic water supply control. The proposed approach is based on continuous observation of the pressure distribution in nodes of the water network. Having a reliable (verified versus field tests) numerical model of the network and its responses for determined inlet and outlet conditions, any modifications to the normal network response (pressure distribution) can be detected. Then, applying proposed bellow numerical procedure, the correction of water supply can be determined.

The proposed methodology is based on so called *Virtual Distortion Method* (VDM) approach, applicable also in the problem of damage identification through monitoring of piezo-generated elastic wave propagation [3]. This technique (called *Piezodiagnostics*) is focused on efficient numerical performance of l inverse, non-linear, dynamic analysis. The crucial point of the concept is pre-computing of structural responses for locally generated impulse loadings by unit virtual distortions (similar to local heat impulses). These responses stored in so-called *influence matrix* allows composing of all possible linear combinations of local non-linearities (due to defect) influence on final structural response. Then, using a gradient-based optimization technique, the intensities of unknown, distributed virtual distortions (modeling local defects) can be tuned to minimize the distance between the computed final structural response and the measured one.

## 2. Definitions and linear analysis

Let us describe the *network analysis* (cf. [1]) based approach to modelling of water systems using oriented *graph* of small example shown in Fig. 1, with topology defined by the following incidence matrix:

$$L = \begin{bmatrix} -1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}, \tag{2.1}$$

where rows correspond to the network's nodes while columns correspond to the branches.

Let us define the following quantities describing the state of the water network:

- $\mathbf{H}$ – the vector of pressure (the height of water potential) in network's nodes,
- $\boldsymbol{\varepsilon}$ – the vector of drop of pressure in network's branches,
- $\mathbf{Q}$ – the vector of water flow in network's branches,
- $\mathbf{R}$ – the vector of flow capacity in network's branches (depends on pipes' cross-sections, length, material, etc.).

Now, the following equations governing the water distribution can be formulated:

- equilibrium of inlets and outlets for nodes:

$$\mathbf{L}\,\mathbf{Q} = -\mathbf{q}, \tag{2.2}$$

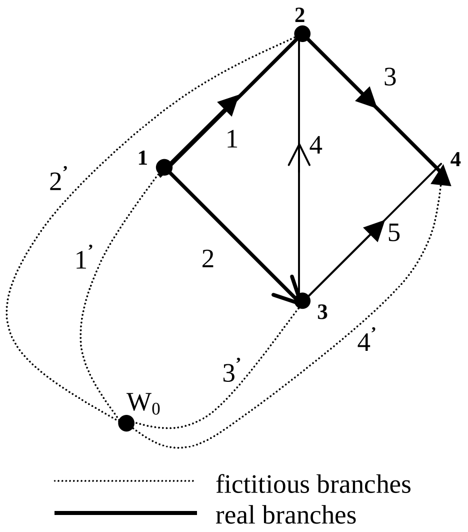


FIGURE 1. Water network.

- definition of drops of pressure for branches:

$$\mathbf{L}^{\mathrm{T}}\,\mathbf{H} = \boldsymbol{\varepsilon}, \tag{2.3}$$

- constitutive relation governing local flow in branches:

$$\mathbf{Q} = -\mathbf{R}\,\boldsymbol{\varepsilon}, \tag{2.4}$$

where $\mathbf{q}$ denotes external inlet to the system.

The constitutive relation (2.3) is non-linear. Nevertheless, let us assume temporarily linearity of this relation. Substituting Eqs. (2.4) and (2.3) to (2.2), the following formula can be obtained:

$$\mathbf{L}\,\mathbf{R}\,\mathbf{L}^{\mathrm{T}}\mathbf{H} = \mathbf{q}, \tag{2.5}$$

allowing determination of water pressure in nodal points as the response for determined inlets.

It will be demonstrated, that having numerical model of the water network and knowing nodal pressure distribution (measured in real time), the optimal water supply can be determined.

## 3. Control of water supply: problem formulation and particular examples

Let us discuss the *control* of water networks. The objective is to minimize the water supply (energy saving) keeping the pressure in all outlets above some limit value. Assuming that the height of outlet nodes can be monitored in real time, specially programmed controller can adapt (feedback) the inlet intensity to meet the minimum supply condition. The aim of the following analysis is to determine the basis for the controller operation.

In the case of low pressure (below the imposed limit value) in any of the outlets, the controller provokes increase of the inlet to achieve the right pressure level. Contrary, in the case of pressure higher than the limit value in all outlets, the controller provokes reduction of the inlet in order to meet the limit-pressure-value in a one (at least) outlet. Let us discuss this problem using the network example illustrated in Fig. 2.

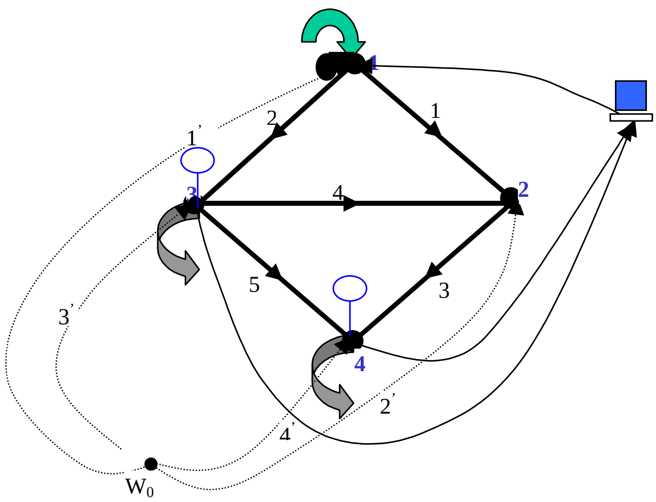


Figure 2. The water supply control concept.

We assume that the heights of nodes No. 3 and No. 4 were previously measured and we have to calculate the unknowns: $R'_3$, $R'_4$, $H_1$, $H_2$ in order to determine outlets $q_3$ and $q_4$. In this case the corresponding set of equation looks as follows:

$$\begin{bmatrix} R_1+R_2 & -R_1 & -R_2 & 0 \\ -R_1 & R_1+R_3+R_4 & -R_4 & -R_3 \\ -R_2 & -R_4 & R_2+R_4+R_5 & -R_5 \\ 0 & -R_3 & -R_5 & R_3+R_5 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} q_1 \\ 0 \\ q_3 \\ q_4 \end{bmatrix} \tag{3.1}$$

where:

$$q_3 = R'_3(H_0 - H_3), \qquad q_4 = R'_4(H_0 - H_4). \tag{3.2}$$

In the above formulas $H$ denotes the water pressure in the node (i.e. the *height* of water), whereas $q$ denotes the flow in the branch. Moreover,

$$R = \frac{K^2}{l},$$

where $K$ is the characteristic of element, and $l$ is the length of element.

Substituting Eqs. (3.2) to (3.1) the following set of equations can be derived:

$$\begin{bmatrix} R_1+R_2 & -R_1 & 0.00 & 0.00 \\ -R_1 & R_1+R_3+R_4 & 0.00 & 0.00 \\ -R_2 & -R_4 & R_3' & 0.00 \\ 0.00 & -R_3 & 0.00 & R_4' \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ R_3' \\ R_4' \end{bmatrix} = \begin{bmatrix} q_1 + R_2 H_3 \\ R_4 H_3 + R_3 H_4 \\ -(R_2+R_4+R_5)H_3 + R_5 H_4 \\ R_5 H_3 + (R_3+R_5) H_4 \end{bmatrix}, \tag{3.3}$$

where it was assumed that the network is supplied only through the node No. 1 (inlet with intensity $q_1$) and the only outlets are through the nodes No. 3 and No. 4 $R_2' = 0$, what means, that the outlet in nodes No. 2 vanishes. The unknowns $H_1$, $H_2$, $R_3'$, $R_4'$ can be determined from Eqs. (3.3) knowing measured $H_3$, $H_4$ and $q_1$.

The problem of active control of the inlet intensity $\gamma\, q_1$ (where $\gamma$ denotes the controlled inlet intensity) can be formulated as follows.

$$\min \gamma, \tag{3.4}$$

subject to the following constraints, requiring that the pressure in the outlet joints is not smaller than some minimal admissible value $H'$:

$$H_3(\gamma\, q_1) \geqslant H', \qquad H_4(\gamma\, q_1) \geqslant H'. \tag{3.5}$$

It can be demonstrated that for the linear case, the above problem formulation leads to the solution requiring that the pressure in the lowest outlet joint $i$ ($H_i = \min$) is equal to the $H'$:

$$H_i = H'. \tag{3.6}$$

Let us now assume that the limit value for the pressure in outlets No. 3 and 4 (Fig. 2) is determined as $H' = 0.80$ and consider the following particular cases.

**Particular case.** Assuming the following data: $R_1$=0.004, $R_2 = R_3 = R_5$=0.016, $R_4$=0.011, $q_1 = 0.05\,\mathrm{m}^3/\mathrm{sec}$, $H_0 = 0.0$ and heaving the following measurements: $H_3 = 0.0438$, $H_4 = 0.0062$ the corresponding set of equations (3.3) takes the following form:

$$\begin{bmatrix} 0.020 & -0.004 & 0.000 & 0.000 \\ -0.004 & 0.031 & 0.000 & 0.000 \\ -0.016 & -0.011 & 0.0438 & 0.000 \\ 0.000 & -0.016 & 0.000 & 0.0062 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ R'_3 \\ R'_4 \end{bmatrix} = \begin{bmatrix} 0.05070 \\ 0.00058 \\ -0.00179 \\ 0.000900 \end{bmatrix}, \tag{3.7}$$

what leads to: $H_1 = 2.606\,\mathrm{m}$ , $H_2 = 0.355\,\mathrm{m}$ , $R'_3 = 1.00$, $R'_4 = 1.00$ and the outlets are as follows: $q_3 = -0.0438\,\mathrm{m}^3/\mathrm{s}$, $q_4 = -0.062\,\mathrm{m}^3/\mathrm{s}$. Note that the minimum height of outlets is $H_4 = H' = 0.0062\,\mathrm{m}$ and no modification of the inlet intensity is required.

**Case I:** when $H_{\min} < H'$. Now let us analyse another case of the above network, when due to the outlets modification the measured heights are: $H_3 = 0.035\,\mathrm{m}$, $H_4 = 0.005\,\mathrm{m}$, what means that: $H_{\min} = H_4 < H'$.

The set of equations (3.3) takes the following form in this case:

$$\begin{bmatrix} 0.020 & -0.004 & 0.000 & 0.000 \\ -0.004 & 0.031 & 0.000 & 0.000 \\ -0.016 & -0.011 & 0.035 & 0.000 \\ 0.000 & -0.016 & 0.000 & 0.005 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ R'_3 \\ R'_4 \end{bmatrix} = \begin{bmatrix} 0.04056 \\ 0.00046 \\ -0.00143 \\ 0.00072 \end{bmatrix}, \tag{3.8}$$

which leads to: $H_1 = 2.0848$, $H_2 = 0.2840$, $R'_3 = 1.00$, $R'_4 = 1.00$ where coefficients $R'_3$ and $R'_4$ determine the current outlet intensities (Eqs. 3.2).

Using determined above quantities of the coefficients $R'_3$ and $R'_4$ we can rearrange the set of equations (3.3) imposing additional condition $H_4 = H'$ and assuming that the inlet intensity $\gamma$ has to be modified (to meet requirement (3.6)):

$$\begin{bmatrix} R_1 + R_2 & -R_1 & -R_2 & q_1 \\ -R_1 & R_1 + R_3 + R_4 & -R_4 & 0.00 \\ -R_2 & -R_4 & R_2 + R_4 + R_5 + R'_3 & 0.00 \\ 0.00 & -R_3 & -R_5 & 0.00 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ \gamma \end{bmatrix} = \begin{bmatrix} 0.00 \\ R_3 H_4 \\ R_5 H_4 \\ -(R_3 + R_5 + R'_4) H_4 \end{bmatrix}. \tag{3.9}$$

Substituting the corresponding values we get:

$$\begin{bmatrix} 0.0200 & -0.004 & -0.0160 & -0.050 \\ -0.004 & 0.0310 & -0.0110 & 0.000 \\ -0.016 & -0.0113 & 1.04303 & 0.000 \\ 0.0000 & -0.0160 & -0.0160 & 0.000 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ \gamma \end{bmatrix} = \begin{bmatrix} 0.00000 \\ 0.000099 \\ 0.000099 \\ -0.00638 \end{bmatrix}, \tag{3.10}$$

which leads to: $H_1 = 2.606\,\mathrm{m}$, $H_2 = 0.355\,\mathrm{m}$, $H_3 = 0.0438\,\mathrm{m}$, $\gamma = 1.25$, and the corresponding outlets: $q_3 = -0.0438\,\mathrm{m}^3/\mathrm{s}$, $q_4 = -0.0062\,\mathrm{m}^3/\mathrm{s}$.

Therefore, we have to increase the inlet by 25% in order to keep the minimal level of height in outlets.

Substituting (just for checking) the modified inlet $\gamma\, q_1$ to the set of equations (3.1) we can get:

$$\begin{bmatrix} 0.0200 & -0.004 & -0.016 & 0.00 \\ -0.004 & 0.0310 & -0.011 & -0.016 \\ -0.016 & -0.0113 & 1.04403 & -0.016 \\ 0.0000 & -0.0160 & -0.0160 & 1.1273 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.00 \\ 0.00 \\ 0.00 \end{bmatrix}, \tag{3.11}$$

which leads to the pressure distribution $H_1 = 2.606\,\mathrm{m}$, $H_2 = 0.355\,\mathrm{m}$, $H_3 = 0.0438\,\mathrm{m}$, $H_4 = 0.05\,\mathrm{m}$ satisfying requirements (3.5).

**Case II:** when $H_{\min} > H'$. In this case we assume that the following heights of nodes 3 and 4: $H_3 = 0.527\,\mathrm{m}$, $H_4 = 0.0074\,\mathrm{m}$ has been observed, what leads to the corresponding set of equations (3.3):

$$\begin{bmatrix} 0.020 & -0.004 & 0.000 & 0.000 \\ -0.004 & 0.031 & 0.000 & 0.000 \\ -0.016 & -0.011 & 0.0527 & 0.000 \\ 0.000 & -0.016 & 0.000 & 0.0074 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ R'_3 \\ R'_4 \end{bmatrix} = \begin{bmatrix} 0.060843 \\ 0.000699 \\ -0.002148 \\ 0.0010812 \end{bmatrix}, \tag{3.12}$$

and to the following result: $H_1 = 3.1265$, $H_2 = 0.4219$, $R'_3 = 1.00$, $R'_4 = 1.00$. The corresponding outlets are $q_3 = -0.009425\,\mathrm{m}^3/\mathrm{s}$, $q_4 = -0.04058\,\mathrm{m}^3/\mathrm{s}$.

The minimum height $H_4$ is bigger than $H'$ what means that the water supply can be reduced. Using determined above quantities of the coefficients $R'_3$ and $R'_4$ we can use the rearranged set of equations (3.9) imposing additional condition $H_4 = H'$ and assuming that the inlet intensity $\gamma$ has to be modified (to meet requirement (3.6)):

$$\begin{bmatrix} 0.0200 & -0.0040 & -0.01600 & -0.060 \\ -0.004 & 0.03100 & -0.01131 & 0.000 \\ -0.016 & -0.0113 & 1.04300 & 0.000 \\ 0.0000 & -0.0160 & -0.0160 & 0.000 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ \gamma \end{bmatrix} = \begin{bmatrix} 0.000000 \\ 0.000099 \\ 0.000099 \\ -0.006391 \end{bmatrix} \tag{3.13}$$

The results are following: $H_1 = 2.626\,\mathrm{m}$, $H_2 = 0.355\,\mathrm{m}$, $H_3 = 0.044\,\mathrm{m}$, $\gamma = 0.83$, and the corresponding outlets $q_3 = -0.044\,\mathrm{m}^3/\mathrm{s}$, $q_4 = -0.0060\,\mathrm{m}^3/\mathrm{s}$. The controller will execute reduction of the inlet intensity by 18% in this case.

Substituting the modified inlet $\gamma\, q_1$ to the set of equations (3.1):

$$\begin{bmatrix} 0.0200 & -0.004 & -0.016 & -0.050 \\ -0.004 & 0.0310 & -0.011 & -0.016 \\ -0.016 & -0.0113 & 0.04677 & -0.016 \\ 0.0000 & -0.0160 & -0.0160 & 0.07258 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.00 \\ 0.00 \\ 0.00 \end{bmatrix}, \tag{3.14}$$

we can check that the resulting pressure distribution: $H_1 = 2.626\,\mathrm{m}$ , $H_2 = 0.355\,\mathrm{m}$, $H_3 = 0.044\,\mathrm{m}$, $H_4 = 0.0062$ satisfy requirements (3.5).

## 4. Conclusions and further steps (non-linear case)

It has been demonstrated (on basis of analysis of particular cases) that optimal control of water supply for the system with linear constitutive equations require solving of two sets of linear equations before each modification of the inlet pressure. However, taking into account real, non-linear constitutive characteristics the optimal control problems become more difficult and cannot be converted just to solving linear equation systems. Nevertheless, further exploitation of analogy between elasto-plastic structures and water networks can be examined. Assuming piecewise-linear constitutive relations describing water flow in the considered networks and introducing concept of *influence matrix*, an analogy to the problem of load minimisation for elasto-plastic structures (with deflections of chosen nodes not lower than some given values) can be explored. In the consequence, an algorithm similar to that for VDM based load maximisation for elasto-plastic truss structures [4] can be expected. This path will be explored in the separate paper.

Local non-linearity can be included into the discussed water network system through so called *virtual distortion* $\varepsilon^0$ introduced into the formula (2.5):

$$\mathbf{L}\,\mathbf{R}(\mathbf{L}^{\mathrm{T}}\mathbf{H} - \boldsymbol{\varepsilon}^0) = \mathbf{q}. \tag{4.1}$$

This concept is analogous to virtual distortions used for simulation of non-linearities in structural systems (cf. [2, 4]). The influence of virtual distortions on the resultant flow redistribution can be calculated making use of so called *influence matrix* $D^H_{ij}$ describing water pressure $H^R_i$ induced in the network as the response for the unit virtual distortion $\varepsilon^0_j = 1$ generated in the branch $j$. Therefore, the vector $\mathbf{H}^R$ can be calculated from the following equation obtained from Eq. (4.1):

$$\mathbf{L\,R\,L}^{\mathrm{T}}\mathbf{H}^{R} = \mathbf{L\,R\,I}. \tag{4.2}$$

Coming back to our small example, let us generate the unit virtual distortion in the branch No. 4. The corresponding set of equations (4.2) with modifications due to the boundary conditions takes the following form:

$$\begin{bmatrix} 0.02 & -0.004 & -0.016 & 0 \\ -0.004 & 0.0313 & -0.0113 & -0.016 \\ -0.016 & -0.0113 & 0.0433 & -0.016 \\ 0 & -0.016 & -0.016 & 1,032 \end{bmatrix} \begin{bmatrix} H_1^{R'} \\ H_2^{R'} \\ H_3^{R'} \\ H_4^{R'} \end{bmatrix} = - \begin{bmatrix} 0 \\ -0.0113\,\varepsilon_4^0 \\ 0.0113\,\varepsilon_4^0 \\ 0 \end{bmatrix}, \tag{4.3}$$

where $\varepsilon_4^0 = 1$.

The resulting distribution of potentials is $\mathbf{H}^R = [0.1508\ \ -0.2513\ 0.2513\ 0.0]^{\mathrm{T}}$. Substituting the resulting distribution of potentials to Eq. (2.3), one can get:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} -0.1508 \\ 0.2513 \\ -0.2513 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.4021 \\ -1,005 \\ -0.2513 \\ 0.50265 \\ 0.2513 \end{bmatrix}, \tag{4.4}$$

what leads to $\varepsilon^R = [0.4021\ \ -0.1005\ \ -0.2513\ 0.50265\ 0.2513]^{\mathrm{T}}$, and constitutes the column of the incidence matrix $\mathbf{D}^{\varepsilon}$. Continuing this procedure for virtual distortions generated in other branches, the full influence matrix can be determined:

$$D_{ij}^{\varepsilon} = \begin{bmatrix} 0.3153 & 0.6847 & -0.288 & 0.4021 & 0.2883 \\ 0.1712 & 0.8288 & 0.072 & -1,005 & -0.072 \\ -0.0721 & 0.0721 & 0.6799 & -0.2513 & 0.3198 \\ 0.1441 & -0.1441 & -0.36 & 0.50265 & 0.3603 \\ 0.072 & -0.072 & 0.6799 & 0.2513 & 0.6801 \end{bmatrix} \tag{4.5}$$

Now, various types of non-linearities can be simulated through properly tuned virtual distortions. Our objective will be focused on simulation of non-linear constitutive characteristics:

$$Q_i = -R_i\,\varepsilon_i^q - R_i \sum_{ij} D_{ij}^{\varepsilon}\,\varepsilon_j^0, \tag{4.6}$$

where $\varepsilon_i^q$ denotes linear response of the system.

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## References

1. N.C. Lind, Analysis of Structures by System Theory, *Journ. Struc. div.*, ASCE 88 ST2 April, 1962.
2. J. Holnicki-Szulc, J.T. Gierlinski, *Structural Analysis, Design and Control by the Virtual Distortion Method*, J.Wiley & Sons, Chichester, 1995.
3. J. Holnicki-Szulc, T.G. Zieliński, New damage identification method through the gradient based optimisation, *Proc. COST International Conference on System Identification & Structural Health Monitoring*, Madrid, 6-9 June, 2000.
4. P. Kolakowski and J. Holnicki-Szulc, Optimal remodelling of truss structures, *Computer Assisted Mechanics and Engineering Sciences*, No. 4, 1997, pp.257-2.